# Archived Seminars 2011/12

October 18, 2011: Oscar Vega (CSUF)

Title: Autotopism Groups in Translation Planes

**Abstract:** A translation plane may be studied in many different ways: geometrically, combinatorially, algebraically, etc. An interesting mix of these three approaches consists in characterizing the geometry of a given plane by assuming some combinatorial information of the collineation group (group of symmetries) of the plane, such as order of a given group, type of action, etc.

In this talk we will see how by assuming a "large enough" (in order) collineation group of the plane (plus extra hypothesis) forces the plane to admit an autotopism group (a group fixing a triangle). This can be used to construct more geometric structures, namely flat flocks of Segre Varieties.

October 25, 2011: Carmen Caprau (CSUF)

Title: Searching for Invariants for Graphs

**Abstract:** Two knots (or graphs) are considered equivalent if one can be transformed into the other via a type of deformation of R3 upon itself. A basic problem of knot theory (or graph theory) is to distinguish between two equivalent knots (or graphs). A useful tool for this recognition problem consists of knot/graph invariants, which are quantities that are the same for equivalent knots/graphs.

The scope of this talk is to construct invariants for trivalent graphs embedded in R3. The first invariant that we will consider is a polynomial invariant which holds only for trivalent graphs whose third of the edges are considered rigid, and it arises from a state model for the 2-variable Kauffman polynomial for knots and links. This was recently constructed by the speaker, in collaboration with James Tipton, a former Fresno State math major. But one should wish for an honest invariant, namely an invariant that deals with topological graphs. So we will discuss ways for improving this polynomial invariant and, also, another idea that we believe will yield an honest numerical invariant for topological trivalent graphs.

November 1, 2011: Tamas Forgacs (CSUF)

Title: On Legendre Multiplier Sequences

Abstract: As part of a program of understanding stability preserving operators, various, well understood polynomial bases serve as starting points for our ongoing investigations. In this talk we discuss multiplier sequences for the Legendre polynomials. These sequences can be viewed as linear operators which preserve the reality of zeros of a given polynomial (when expanded in the Legendre basis). We characterize linear, quadratic and geometric Legendre multiplier sequences, and discuss some interesting phenomena occurring with these polynomials that we haven't seen in the literature before. This is joint work with 3 students: K. Blakeman (LMU), E. Davis (BYU) and K. Urabe (Fresno State).

## March 2, 2012: Megan Kuneli, Robyn McDonald, Jamie Peabody, Karen Willis, Katherine Urabe (CSUF)

Title: Three Presentations by Undergraduate Students

#### Abstracts:

1. Megan Kuneli and Robyn McDonald: The well-covered dimension of products of graphs.

Abstract: Our project focuses on the well-covered dimension of (Cartesian) products of graphs, specifically paths and cycles. We have proved that the well-covered dimension of these products is always zero when the paths and/or cycles are sufficiently large. We have also studied product of graphs in other families. By doing this we have found an example of a graph that has dimension depending on the characteristic of the field used to define the vector space of weights. We have also found a formula for finding the well-covered dimension of the product of complete graphs.

2. Jamie Peabody and Karen Willis: Grim: A subtraction game on graphs.

Abstract: We study winning strategies for the game Grim, which is a subtraction game played on a finite graph. A legal move in this game consists of a player removing a vertex and consequently all edges attached to the vertex. A player wins if they can leave the other player with no legal moves (when there are no more vertices left, or all remaining vertices are disconnected). This game seems to have not been studied before. Our focus has been on winning strategies on several different families of graphs. We have found some fascinating strategies for complete graphs, complete bipartite graphs, some products of graphs, and any symmetric graph. We also have some results for paths, cycles, and wheels.

3. Katherine Urabe: On Legendre Multiplier Sequences.

Abstract: Let  $B = b_k(x)$  be a simple set of polynomials. A sequence of real numbers  $G = g_k$  is called a B-multiplier sequence if given any polynomial  $p(x) = a_n b_n(x) + \dots + a_1 b_1(x) + a_0 b_0$  with only real zeros, the polynomial  $G[p(x)] = a_n g_n b_n(x) + \dots + a_1 g_1 b_1(x) + a_0 g_0 b_0$  also has only real zeros. Such sequences have been completely characterized in case B is the standard basis (Polya-Schur, 1914), or the set of Hermite polynomials (Piotrowski, 2007). Good progress has been made recently in the case when B is the set of generalized Laguerre polynomials (Forgacs and Piotrowski, 2011) although the characterization of generalized Laguerre multiplier sequences is still incomplete. In this talk we describe multiplier sequences for the set of Legendre polynomials, and discuss how Legendre multiplier sequences are related to the classical, Hermite and Laguerre multiplier sequences.

#### April 16, 2012: Carmen Caprau, Assistant Professor (CSUF)

Title: An Invariant for Handlebody-Tangles I

**Abstract:** The aim of the talk is to construct a numerical invariant for *handlebody-tangles* coming from the well-known *Kauffman bracket* invariant for knots and links. I will start by

giving the necessary background (some of which appears below), after which I will explain how to construct a 3-move invariant for (m, n)-tangles by employing the *skein* (m, n)-module corresponding to a fix non-zero complex number q. Then we extend this to an invariant for *trivalent tangles*, which automatically yields an invariant for handlebody-tangles.

A handlebody-tangle is a disjoint union of handlebodies embedded in the 3-ball  $B^3$ , such that the intersection of the handlebodies with  $\partial B^3$  consists of disks, called end disks. Two handlebody-tangles are equivalent if one can be transformed into the other by an isotopy of  $B^3$  which fixes  $\partial B^3$ . A handlebody-link/knot can be regarded as a handlebody-tangle with no end disks. A trivalent tangle is a uni-trivalent graph embedded in  $B^3$  whose intersection with  $\partial B^3$  consists of all of its univalent vertices, called end points. A trivalent tangle with no end points is a knotted trivalent graph.

Any handlebody-link (or handlebody-tangle) is a regular neighborhood of some knotted trivalent graph (or trivalent tangle), thus there is a one-to-one correspondence between the set of handlebody-links (or handlebody-tangles) and that of neighborhood equivalence classes of spatial trivalent graphs (or trivalent tangles). Therefore, we can study handlebody-tangles through diagrams of trivalent tangles. Since a genus one handlebody-knot is a regular neighborhood of a knot, handlebody-knot theory is a generalization of knot theory.

April 23, 2012: Carmen Caprau, Assistant Professor (CSUF)

Title: An Invariant for Handlebody-Tangles II

**Abstract:** (Continued from Last Week)

April 27, 2012: Stefaan Delcroix, Associate Professor (CSUF)

Title: Haussdorf Dimensions of Cartesian and Tensor Products of Graphs and Fractals

Abstract: We start with the definition of the cartesian product and the tensor product of graphs (note that these graphs can be weighted or unweighted, directed or undirected). The main result is the relation between the eigenvalues of a product of graphs and the eigenvalues of the original graphs. Next, we introduce the concept of the Haussdorf dimension of a fractal and the relation with the eigenvalues of a strongly connected, weighted, directed graph. Finally, we use the Perron-Frobenius Theorem to calculate the Haussdorf dimension of cartesian and tensor products of graphs.

April 30, 2012: David Heywood and Dionne Ibarra (CSUF)

Title: Jaeger's Model for the Two-Variable Kauffman Polynomial

**Abstract:** The two-variable Kauffman polynomial and the HOMFLY- PT polynomial are two beautiful invariants for links. In the last 20 years, or so, they have been extensively studied. A remarkable result by Fracois Jaeger relates these, very distinct invariants, by presenting the Kauffman polynomial of an unoriented link L as a weighted sum of HOMFLY-PT polynomials of oriented links associated to L.

The goal of this talk is to describe this work by Jaeger.

#### May 4, 2012: Oscar Vega, Assistant Professor (CSUF)

#### Title: Embeddings of Graphs in Projective Spaces

**Abstract:** Embeddings of geometric structures into other projective planes have been studied for a long time, mostly since Erdos conjectured (in 1979) that every partial linear space can be embedded in a projective plane. This question had already been settled by Hall for infinite planes back in 1943, but the problem is still open for finite planes.

Most of this talk will be a friendly introduction to the objects, techniques, and ideas used, in recent work by F. Lazebnik, K. Mellinger and the speaker, to study embeddings of graphs in projective planes. In the final part of the talk, a preview of the problem that students attending our Summer REU will work on (embedding of graphs in projective spaces) will be given.

### May 7, 2012: Joel Smith (CSUF)

#### Title: The HOMFLY-PT Polynomial of a Braid

**Abstract:** We construct a representation of the Artin braid group  $B_n$  into a certain algebra, and then apply a trace function associating a Laurent polynomial P(b) to a braid b in  $B_n$ . This polynomial is invariant under equivalences and conjugation in  $B_n$ , and under Markov's moves.

This construction allows us to recover the HOMFLY-PT polynomial of a link L from the braid polynomial P(b), whenever L is the "trace closure" of b.